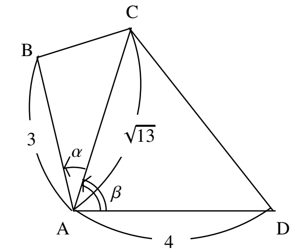
$\boxed{9}$



$BC^2 + CD^2 = 25$

Set $\angle BAC = \alpha$, $\angle CAD = \beta$. Then by means of second cosine formula.

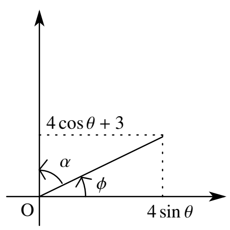
$$\begin{cases} BC^2 = 3^2 + AC^2 - 2 \cdot 3 \cdot AC \cdot \cos\alpha \\ CD^2 = 4^2 + AC^2 - 2 \cdot 4 \cdot AC \cdot \cos\beta \end{cases} \qquad \cdots ①$$

Here $BC^2 + CD^2 = 25$,then by means of ①.

$$AC = 3\cos\alpha + 4\cos\beta \qquad \cdots ②$$

Set $\angle BAD = \theta$, then.

$$\begin{aligned} AC &= 3\cos\alpha + 4\cos(\theta - \alpha) \\ &= 3\cos\alpha + 4(\cos\theta\cos\alpha + \sin\theta\sin\alpha) \\ &= 4\sin\theta\sin\alpha + (4\cos\theta + 3)\cos\alpha \\ &= \sqrt{25 + 24\cos\theta}\ \sin(\alpha + \phi) \end{aligned}$$



$$\left(\sin\phi = \frac{4\cos\theta + 3}{\sqrt{25 + 24\cos\theta}},\ \cos\phi = \frac{4\sin\theta}{\sqrt{25 + 24\cos\theta}} \right)$$

Therefore

$$AC \leq \sqrt{25 + 24\cos\theta} \text{ (be equal when } \alpha + \phi = 90^\circ \text{)} \qquad \cdots ③$$

$AC = \sqrt{13}$, then by means of ③.

$$\sqrt{25 + 24\cos\theta} \geq \sqrt{13} \longleftrightarrow \cos\theta \geq -\frac{1}{2}$$

$$\therefore \quad 0^\circ < \theta \leq 120^\circ < 180^\circ \qquad \cdots ④$$

In addition, if $\theta = 120^\circ$, then $0^\circ < \phi < 90^\circ$. Thus α such that $\alpha + \phi = 90^\circ$ can exists. Therefore the absolute maximum of θ is.

$$\theta = 120^\circ$$

Comment

If $\theta = 120^\circ$,then the length of segments BC and CD are 4, 3 respectively.