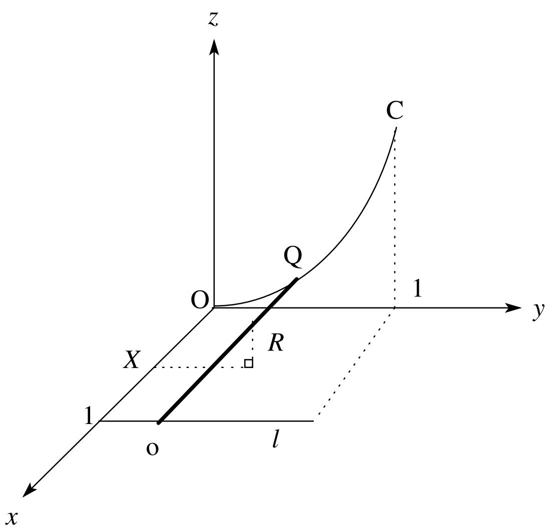
$\boxed{8}$



Cordinates of P and Q are as follows respectively.

$$\overrightarrow{OP} = \begin{pmatrix} 1 \\ s \\ 0 \end{pmatrix}$$

$$\overrightarrow{OQ} = \begin{pmatrix} 0 \\ t \\ t^2 \end{pmatrix}$$

Next, take a point R on the segment PQ with x value of X. Then R divides the segment PQ in the ratio of $X : (1-X)$, so

$$\begin{aligned}\overrightarrow{OR} &= (1-X)\overrightarrow{OQ} + X\overrightarrow{OP} \\ &= \begin{pmatrix} X \\ t(1-X) + sX \\ t^2(1-X) \end{pmatrix}\end{aligned}$$

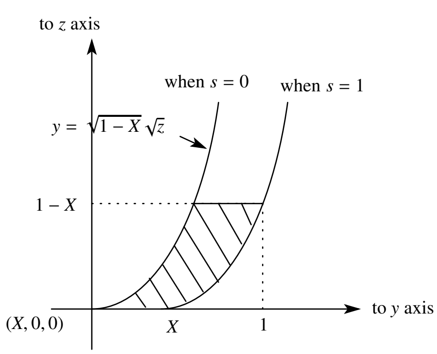
Therfore y and z cordinates of R are as follows respectively.

$$\begin{cases} y &= t(1-X) + sX \\ z &= t^2(1-X) \end{cases}$$

Eliminate t considering $0 \leq t \leq 1$.

$$y = \sqrt{1-X} \cdot \sqrt{z} + sX \ (0 \leq z \leq 1-X) \qquad \cdots ①$$

Here, if s changes from 0 to 1, the area of the curve ① passes on the plane $x = X$ is as same the area which $y = \sqrt{1-X} \cdot \sqrt{z}$ $(0 \leq z \leq 1-X)$ translates by X to the direction of y axis. Therefore, the cross section of K by the plane $x = X$ is as follows.



cross section by $x = X$

Thus, the area of the cross section is.

$$\begin{aligned}S &= \int_0^{1-X} \left\{ (\sqrt{1-X} \cdot \sqrt{z} + X) - (\sqrt{1-X} \cdot \sqrt{z}) \right\} dz \\ &= \int_0^{1-X} X dz = X(1-X)\end{aligned}$$

Therefore the volume of K is

$$V = \int_0^1 x(1-x)dx = \frac{1}{6} \qquad \cdots ans.$$

Comment

Rough sketch of K