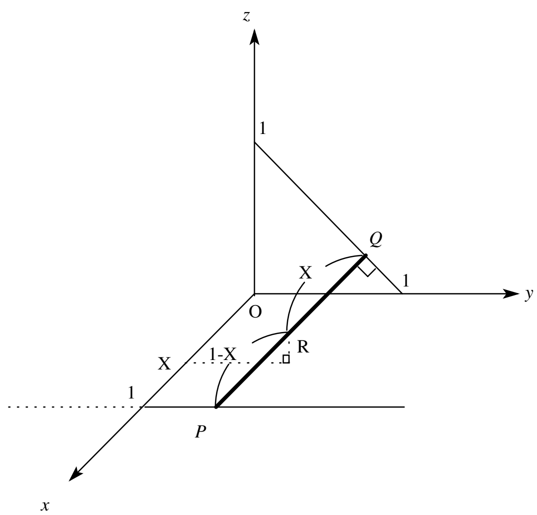
7



Cordinates of P and Q are as follows respectively.

$$\overrightarrow{OP} = \begin{pmatrix}1\\0\\0\end{pmatrix} + t\begin{pmatrix}0\\1\\0\end{pmatrix}$$

$$\overrightarrow{OQ} = \begin{pmatrix}0\\0\\1\end{pmatrix} + s\begin{pmatrix}0\\1\\-1\end{pmatrix}$$

Therefore

$$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP} = \begin{pmatrix}-1\\s-t\\1-s\end{pmatrix}$$

Because $\overrightarrow{PQ} \perp m$

$$\overrightarrow{PQ} \cdot \begin{pmatrix}0\\1\\-1\end{pmatrix} = 2s - t - 1 = 0$$

$$\therefore\ t = 2s - 1 \qquad \cdots①$$

Next, take a point R in the segment PQ with an x value of X, then R devide the segment PQ in the ratio of $X : (1 - X)$ that,

$$\overrightarrow{OR} = (1 - X)\overrightarrow{OQ} + X\overrightarrow{OP} = \begin{pmatrix}X\\X(t-s)+s\\(X-1)(s-1)\end{pmatrix} = \begin{pmatrix}X\\X(s-1)+s\\(X-1)(s-1)\end{pmatrix} \qquad \cdots②$$

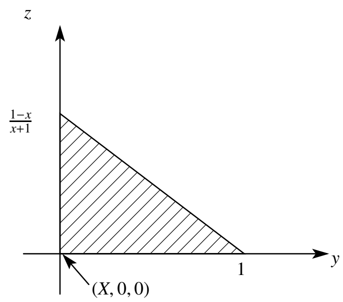
Thus y and z values of R are as follows respectively.

$$\begin{cases} y &= X(s-1)+s \\ z &= (X-1)(s-1) \end{cases}$$

Eliminate s.

$$z = \frac{1-X}{X+1} \cdot (1-y)$$

Thus, the cross section of K by a plane $x = X$ is as follows.



Therefore, the volume of K is.

$$\begin{aligned} V &= \frac{1}{2}\int_0^1 \frac{1-x}{x+1}dx \\ &= \frac{1}{2}\int_0^1 \left(\frac{2}{x+1} - 1\right)dx \\ &= \frac{1}{2}\Big[2\log(x+1) - x\Big]_0^1 \\ &= \log 2 - \frac{1}{2} \end{aligned} \qquad \cdots ans.$$

Comment

Rough sketch of K is as follows.