

6 Same as problem 5 halfway. P and Q are crossings of m and a curved surface of the cylinder. As same as problem 5,

$$PQ = \frac{2}{\sqrt{3}}|2 \sin \theta - 1| \quad \dots \textcircled{1}$$

Next, let π be a tangent plane to the cylinder at a line PQ

$$\pi : \cos \theta y + \sin \theta z = 1 \quad \dots \textcircled{2}$$

Let h to be a distance between π and $A(0, 0, 2)$, then

$$h = \frac{|2 \sin \theta - 1|}{\sqrt{\cos^2 \theta + \sin^2 \theta}} = |2 \sin \theta - 1| \quad \dots \textcircled{3}$$

Define plane $\pi(\theta)$ as a plane which includes a line $m(\theta)$ and point $A(0, 0, 2)$. In addition, define the volume of the cone enclosed by $\pi(\theta)$, $\pi(\theta + \Delta)$ and the curved surface of the cylinder as ΔV , then if $\Delta\theta \neq 0$

$$\Delta V \doteq \frac{1}{3} \times (PQ \times 1 \cdot \Delta\theta) \times h = \frac{2}{3\sqrt{3}}(2 \sin \theta - 1)^2 \Delta\theta \quad \dots \textcircled{4}$$

Therefore the volume of the cone above the cylinder is

$$\begin{aligned} V &= \frac{2}{3\sqrt{3}} \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} (2 \sin \theta - 1)^2 d\theta \\ &= \frac{4\sqrt{3}}{9} \pi - 2 \quad \dots \text{ans.} \end{aligned}$$

