5 m is a line which passes a point$(\cos\theta, \sin\theta)$ on the curved surface of the cylinder and parallel to x axis.Then

$$m; \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ \cos\theta \\ \sin\theta \end{pmatrix} + t \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad \cdots ①$$

The equation of the curved surface of the cylinder is

$$3(x^2 + y^2) = (z - 2)^2 \qquad \cdots ②$$

Solve equation① and equation②

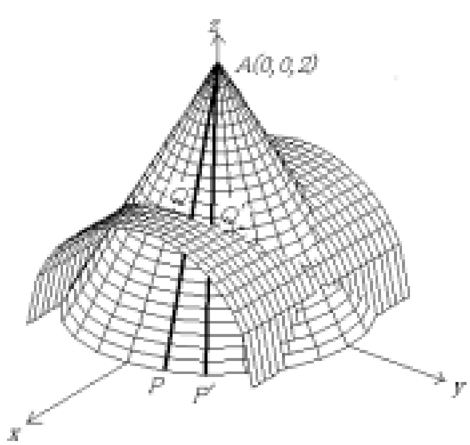
$$\begin{aligned} 3(t^2 + \cos^2\theta) &= (\sin\theta - 2)^2 \\ 3t^2 &= \sin^2\theta - 4\sin\theta + 4 - 3(1 - \sin^2\theta) \\ &= 4\sin^2\theta - 4\sin\theta + 1 \\ &= (2\sin\theta - 1)^2 \end{aligned}$$

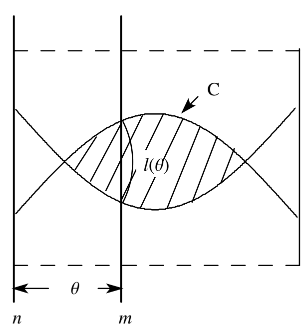
Therefore, let the distance between two crossings of ① and ② be $l(\theta)$, then

$$l(\theta) = 2|t| = \frac{2}{\sqrt{3}}|2\sin\theta - 1| \qquad \cdots ③$$

Then the angle of rotation between $m(\theta)$ and $n; y = 1, z = 0$ around x axis is θ, thus when we develop the curved surface, the distance between $m(\theta)$ and n is $1 \times \theta = \theta$. Because of that , the net of the cylinder $(z \geq 0)$ is like a figure. Therefor the area is

$$\begin{aligned} S &= \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} l(\theta)d\theta \\ &= \frac{2}{\sqrt{3}} \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} (2\sin\theta - 1)d\theta \\ &= 4 - \frac{4\sqrt{3}}{9}\pi \qquad \cdots ans. \end{aligned}$$





net of the cylinder