$\boxed{4}$ $P(t, 0, \sqrt{t})$ is a point in the curve C and $Q(x, y, 0)$ is the tip of the brush. Because $PQ = 1$,

$$(x-t)^2 + y^2 + (\sqrt{t})^2 = 1$$
$$\therefore t^2 + (-2x+1)t + x^2 + y^2 - 1 = 0 \qquad \cdots ①$$

Because $t \geq 0$, E is the area of (x, y) in which equation ① has more than one non negative solutions. Let difine $f(t)$ as follows.

$$f(t) = t^2 + (-2x+1)t + x^2 + y^2 - 1$$
$$= \left(t - \frac{2x-1}{2}\right)^2 + x + y^2 - \frac{5}{4}$$

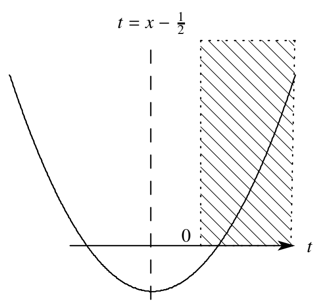
Then the axis of $Y = f(t)$ is $t = \dfrac{2x-1}{2}$

Therefore, the conditions in which the equation ① has more than one non negative solutions are as follows.

(i)when $x \leq \dfrac{1}{2}$.

$f(0) = x^2 + y^2 - 1 \leq 0$

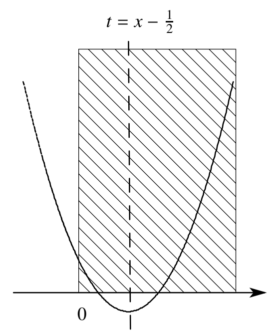
$\longleftrightarrow x^2 + y^2 \leq 1$



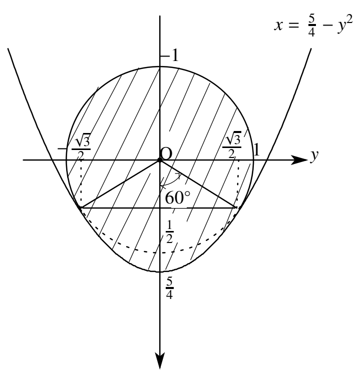
(ii) when $x > \dfrac{1}{2}$.

$f(x - \frac{1}{2}) = x + y^2 - \dfrac{5}{4} \leq 0$

$\longleftrightarrow x \leq \dfrac{5}{4} - y^2$



$x^2 + y^2 = 1$ and $x = \dfrac{5}{4} - y^2$ contact at $\left(\dfrac{1}{2}, \pm\dfrac{\sqrt{3}}{2}\right)$, E is the shaded portion of the figure at lower left.



Thus,the area of E is;

$$S = \pi \cdot 1^2 \cdot \frac{240°}{360°} + \frac{1}{2} \cdot \frac{\sqrt{3}}{2} + \int_{-\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} \left\{\left(\frac{5}{4} - y^2\right) - \frac{1}{2}\right\} dy$$
$$= \frac{2\pi}{3} + \frac{\sqrt{3}}{4} + \frac{1}{6}\left(\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}\right)^3$$
$$= \frac{2\pi}{3} + \frac{3\sqrt{3}}{4} \qquad \cdots ans.$$