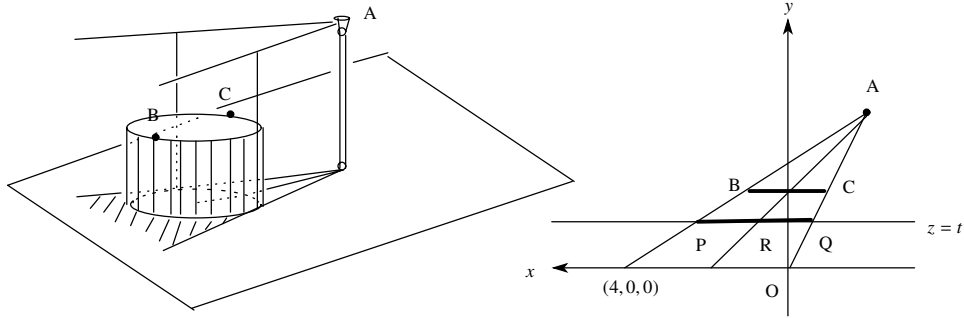
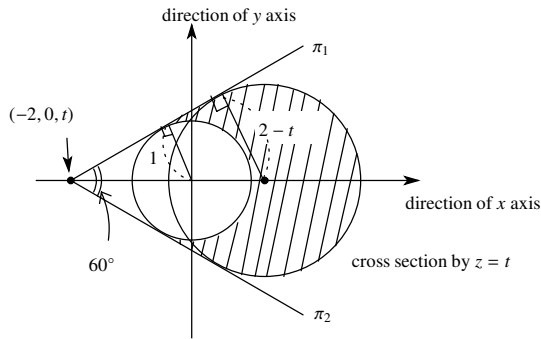


3



Take the origin at the center of the base of  $D$ , and set  $x, y$  axes on the ground such that the tip of the streetlight has coordinates  $(-2, 0, 2)$ . Take two points  $B$  and  $C$ , which have coordinates  $(1, 0, 1)$  and  $(-1, 0, 1)$  respectively. In addition, take two tangent planes to  $D$  through  $A$  named  $\pi_1, \pi_2$ . Then,  $\pi_1, \pi_2$  divide the space into four parts, but only the part which includes  $D$  has no light.

Second, think about the cross section with the shadow produced by upper base of  $D$  ( $x^2 + y^2 \leq 1, z = 1$ ) and a plane  $z = t$  ( $0 \leq t \leq 1$ ). It is inside the circle of radius  $2-t$ , with the center  $R(2(t-1), 0, t)$  excluding  $D$ .



Therefore, the cross section with the shadow produced by  $D$  and the plane  $z = t$  ( $0 \leq t \leq 1$ ) becomes like the figure at left. Let the area of the cross section be  $S(t)$ , then

$$\begin{aligned}
 S(t) &= \pi(2-t)^2 \times \frac{240^\circ}{360^\circ} + \{1 + (2-t)\} \times \sqrt{3}(1-t) - \pi \times 1^2 \times \frac{240^\circ}{360^\circ} \pi \\
 &= \frac{1}{3}(2t^2 - 8t + 6)\pi + \sqrt{3}(t^2 - 4t + 3)
 \end{aligned}$$

Thus the volume of the shadow is;

$$\begin{aligned}
 V &= \int_1^0 \left\{ \frac{1}{3}(2t^2 - 8t + 6)\pi + \sqrt{3}(t^2 - 4t + 3) \right\} dt \\
 &= \frac{8}{9}\pi + \frac{4}{3}\sqrt{3}
 \end{aligned}$$

... ans.