16

Case(i) If $(*)$ has imaginary number solutions. We can set three solutions such as:

$$\begin{cases} \alpha = p + qi \\ \beta = p - qi \\ \gamma = r \end{cases} \qquad (p, q, r \text{are real numbers})$$

Then, by means of the formula between solutions and coefficients.

$$\begin{cases} (p + qi) + (p - qi) + r = 0 \\ (p + qi)(p - qi) + r(p + qi) + r(p - qi) = -3 \end{cases} \iff \begin{cases} 2p + r = 0 \\ p^2 - \dfrac{q^2}{3} = 1 \end{cases} \qquad \cdots ①$$

If we define a function $f(x) = -x^3 + 3x$ (x is real), real number solutions of $(*)$ are the values of x where two graphs i.e. $y = f(x)$ and $y = k$ intersect. Therefore, by means of a graph,

$$r < -2$$

Thus.

$$|\gamma + 2| - |\gamma - 2| = |r + 2| - |r - 2| = -(r + 2) + (r - 2) = -4 \qquad \cdots ②$$

In addition, by means of ①, imaginary solutions of $(*)$ are in the hyperbola C on the $p - q$ plane.

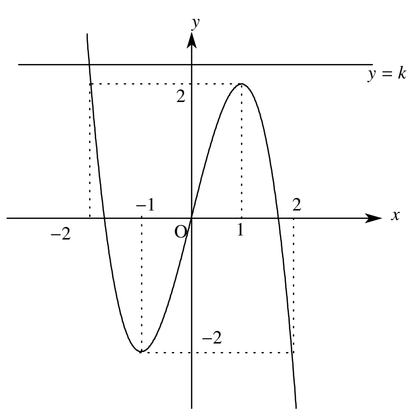
$$C;\, p^2 - \frac{q^2}{3} = 1 \ (p > 1)$$

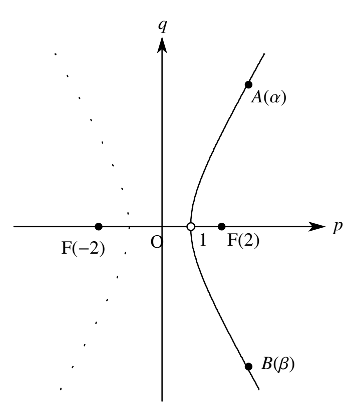
Focuses of C are $(\pm 2, 0)$, then.

$$|\alpha + 2| - |\alpha - 2| = |\beta + 2| - |\beta - 2| = 2 \qquad \cdots ③$$

Because of ② and ③.

$$|\alpha + 2| + |\beta + 2| + |\gamma + 2| - |\alpha - 2| - |\beta - 2| - |\gamma - 2| = 2 + 2 + (-4) = 0$$





Case(ii) If $(*)$ has three real number solutions.

$$-2 \leq \alpha, \beta, \gamma \leq 2$$

Therefore,

$$\begin{aligned}
&|\alpha + 2| + |\beta + 2| + |\gamma + 2| - |\alpha - 2| - |\beta - 2| - |\gamma - 2| \\
&= (\alpha + 2) + (\beta + 2) + (\gamma + 2) + (\alpha - 2) + (\beta - 2) + (\gamma - 2) \\
&= 2(\alpha + \beta + \gamma) = 0
\end{aligned}$$

To conclude, because of (i) and (ii).

$$|\alpha + 2| + |\beta + 2| + |\gamma + 2| - |\alpha - 2| - |\beta - 2| - |\gamma - 2| = 0 \qquad \cdots ans.$$