

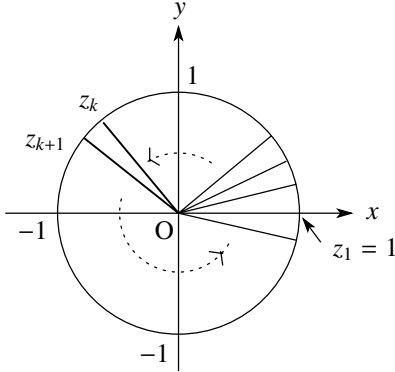
$$z_k = \cos \frac{2\pi}{n}(k-1) + i \sin \frac{2\pi}{n}(k-1)$$

Therefore,

$$\begin{aligned} \sum_{k=1}^n \frac{z_{k+1} - z_k}{z_k} &= \sum_{k=1}^n \left(\frac{z_{k+1}}{z_k} - 1 \right) \\ &= \left(\frac{\cos \frac{2\pi}{n}k + i \sin \frac{2\pi}{n}k}{\cos \frac{2\pi}{n}(k-1) + i \sin \frac{2\pi}{n}(k-1)} - 1 \right) \\ &= \sum_{k=1}^n \left(\cos \frac{2\pi}{n} + i \sin \frac{2\pi}{n} - 1 \right) \\ &= n \left(\cos \frac{2\pi}{n} + i \sin \frac{2\pi}{n} - 1 \right) \quad \dots \textcircled{1} \end{aligned}$$

Set $\frac{2\pi}{n} = \theta$, then if $n \rightarrow \infty$ then $\theta \rightarrow 0$ so,

$$\begin{aligned} I &= \lim_{n \rightarrow \infty} n \left(\cos \frac{2\pi}{n} + i \sin \frac{2\pi}{n} - 1 \right) \quad \leftarrow \textcircled{1} \\ &= 2\pi \left(\lim_{\theta \rightarrow 0} \frac{\cos \theta - 1}{\theta} + i \lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} \right) \\ &= 2\pi i \quad \dots \text{ans.} \end{aligned}$$



Next,

$$\sum_{k=1}^n \frac{z_{k+1} - z_k}{z_k^2} = \sum_{k=1}^n \frac{1}{z_k} \left(\frac{z_{k+1}}{z_k} - 1 \right) = \left(\cos \frac{2\pi}{n} + i \sin \frac{2\pi}{n} - 1 \right) \sum_{k=1}^n \frac{1}{z_k} \quad \dots \textcircled{2}$$

Here,

$$\sum_{k=1}^n \frac{1}{z_k} = \sum_{k=1}^n \left\{ \cos \frac{2\pi}{n}(k-1) - i \sin \frac{2\pi}{n}(k-1) \right\} = \sum_{k=1}^n \left(\cos \frac{2\pi}{n} - i \sin \frac{2\pi}{n} \right)^{k-1}$$

The right side is a sum of the first n th terms of the sequence, such that the first term is 1, and the common ratio is $\cos \frac{2\pi}{n} - i \sin \frac{2\pi}{n}$. If we set $r = \cos \frac{2\pi}{n} - i \sin \frac{2\pi}{n}$, then.

$$r^n = \left(\cos \frac{2\pi}{n} - i \sin \frac{2\pi}{n} \right)^n = \cos 2\pi - i \sin 2\pi = 1$$

Thus, if $n \geq 2$

$$\sum_{k=1}^n \frac{1}{z_k} = \frac{1-r^n}{1-r} = 0$$

Therefore, because of ②

$$J = 0$$

... ans.