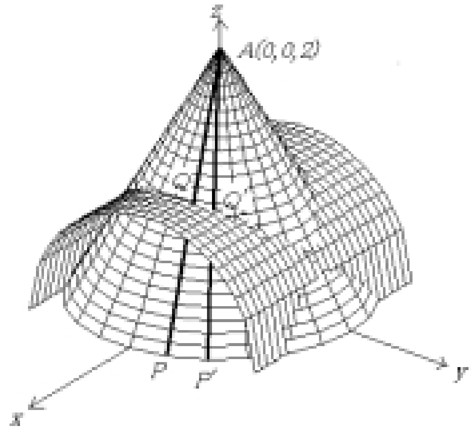
$\boxed{11}$ Take a point $P\left(\frac{2}{\sqrt{3}}\cos\theta, \frac{2}{\sqrt{3}}\sin\theta, 0\right)$, in the curve $x^2+y^2=\frac{4}{3}, z=0$. The line AP and the clinder intersects at more than one points. From those crossings, we take the closer point looking from the point A and name it $Q(x,y,z)$. Then, Q is in the line AP.



$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \overrightarrow{OA} + t\overrightarrow{AP} = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} + t\begin{pmatrix} \frac{2}{\sqrt{3}}\cos\theta \\ \frac{2}{\sqrt{3}}\sin\theta \\ -2 \end{pmatrix} \qquad \cdots ①$$

Q is also on the curved surface of the cylinder,then.

$$y^2+z^2=1 \qquad \cdots ②$$

By means of ①,②

$$\left(t\cdot\frac{2}{\sqrt{3}}\sin\theta\right)^2 + (2-2t)^2 = 1$$

$$\therefore\ (4\sin^2\theta+12)t^2 - 24t + 9 = 0$$

$$\therefore\ t = \frac{12 \pm \sqrt{144-9(4\sin^2\theta+12)}}{4\sin^2\theta+12} = \frac{12 \pm \sqrt{36(1-\sin^2\theta)}}{4\sin^2\theta+12} = \frac{12 \pm 6|\cos\theta|}{4\sin^2\theta+12}$$

The area of the curved surface of the cone above the cylinder is four times as big as the area included in $x \geq 0, y \geq 0$.Then we can assume $0 \leq \theta \leq \frac{\pi}{2}$,on that conditions,

$$t = \frac{12-6\cos\theta}{4\sin^2\theta+12} = \frac{3}{2(2+\cos\theta)} \qquad \cdots ③$$

Therfore,

$$\begin{aligned} \left|\overrightarrow{AQ}\right| &= |t|\cdot\sqrt{\left(\frac{2}{\sqrt{3}}\cos\theta\right)^2 + \left(\frac{2}{\sqrt{3}}\sin\theta\right)^2 + (-2)^2} \\ &= \frac{4\sqrt{3}}{3}|t| \\ &= \frac{4\sqrt{3}}{3}\cdot\frac{3}{2(2+\cos\theta)} \\ &= \frac{2\sqrt{3}}{2+\cos\theta} \end{aligned} \qquad \cdots ④$$

The ratio between the slanted height and the radius of the base of the cone is $2:1$, thus, if $\angle POP' = \Delta\theta$ then $\angle QAQ' = \frac{1}{2}\Delta\theta$. Therfore, the area aquired is.

$$\begin{aligned} S &= 4\times\frac{1}{2}\int_0^{\frac{\pi}{2}} \left|\overrightarrow{AQ}\right|^2 \frac{1}{2}d\theta \\ &= 12\int_0^{\frac{\pi}{2}} \frac{1}{(2+\cos\theta)^2}d\theta \end{aligned} \qquad \cdots ⑤$$

We use the next expression to find the integration above.

$$\left(\frac{\sin\theta}{2+\cos\theta}\right)' = \frac{2\cos\theta+1}{(2+\cos\theta)^2} \qquad \cdots(*)$$

We define I and J as follows.

$$I = \int_0^{\frac{\pi}{2}} \frac{1}{(2+\cos\theta)^2}d\theta, \quad J = \int_0^{\frac{\pi}{2}} \frac{\cos\theta}{(2+\cos\theta)^2}d\theta$$

Then by means of $(*)$.

$$2J + I = \int_0^{\frac{\pi}{2}} \frac{2\cos\theta+1}{(2+\cos\theta)^2}d\theta = \left[\frac{\sin\theta}{2+\cos\theta}\right]_0^{\frac{\pi}{2}} = \frac{1}{2} \qquad \cdots⑥$$

In addition,

$$2I + J = \int_0^{\frac{\pi}{2}} \frac{2+\cos\theta}{(2+\cos\theta)^2}d\theta = \int_0^{\frac{\pi}{2}} \frac{1}{2+\cos\theta}d\theta$$

We define $\tan\frac{\theta}{2} = t$, then

$$\cos\theta = \frac{1-t^2}{1+t^2}, \quad d\theta = \frac{2}{1+t^2}dt, \quad \begin{array}{c|ccc} \theta & 0 & \to & \frac{\pi}{2} \\ \hline t & 0 & \to & 1 \end{array}$$

$$\therefore\ 2I + J = \int_0^1 \frac{2}{t^2+3}dt$$

Again, we define $t = \sqrt{3}\tan u$, then

$$dt = \frac{\sqrt{3}}{\cos^2 u}du, \quad \begin{array}{c|ccc} t & 0 & \to & 1 \\ \hline u & 0 & \to & \frac{\pi}{6} \end{array}$$

$$\therefore\ 2I + J = \int_0^{\frac{\pi}{6}} \frac{2}{3(1+\tan^2 u)} \cdot \frac{\sqrt{3}}{\cos^2 u}du = \frac{2\sqrt{3}}{3}\int_0^{\frac{\pi}{6}} du = \frac{\sqrt{3}}{9}\pi \qquad \cdots⑦$$

Using ⑥ and⑦

$$I = \frac{2\sqrt{3}}{27}\pi - \frac{1}{6}$$

Substitute it to ⑤.

$$S = 12\left(\frac{2\sqrt{3}}{27}\pi - \frac{1}{6}\right) = \frac{8\sqrt{3}}{9}\pi - 2 \qquad \cdots ans.$$