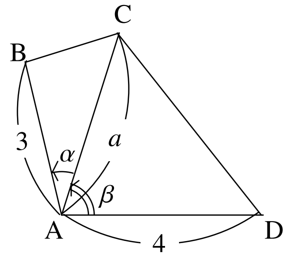
10



$BC^2 + CD^2 = 25$

Much the same as problem 9 till ③. Because AC=a then by means of ③.

$$\sqrt{25 + 24\cos\theta} \geq a \iff \cos\theta \geq \frac{a^2 - 25}{24}$$

If, $1 < a < 7$, $-1 < \dfrac{a^2 - 25}{24} < 1$, then

$$0^\circ < \theta \leq \theta_0 < 180^\circ \left(\cos\theta_0 = \frac{a^2 - 25}{24}\right) \qquad \cdots ④$$

In addition, because $4\sin\theta_0 > 0$, so $-90^\circ < \phi < 90^\circ$.Then α such that $\alpha + \phi = 90^\circ$ can exists.

Therfore, the absolute maximum of θ is.

$$\theta = \theta_0$$

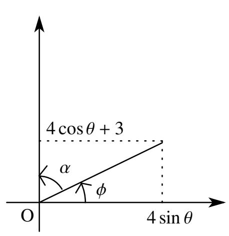
Then, $\cos\alpha = \cos(90^\circ - \phi) = \sin\phi = \dfrac{4\cos\theta_0 + 3}{\sqrt{25 + 24\cos\theta_0}} = \dfrac{a^2 - 7}{6a}$

$$BC^2 = 3^2 + a^2 - 2 \cdot 3 \cdot a \cdot \frac{a^2 - 7}{6a} = 16$$

$$\therefore \quad \begin{cases} BC = 4 \\ CD = 3 \end{cases} \qquad \cdots ans.$$

In addition, if $\angle BAD$ has absolute maximum.

$$\cos\angle BAD = \frac{a^2 - 25}{24}$$



Comment

$$\begin{cases} \text{If, } a \geq 7, & \text{no quadrilateral} \\ \text{If, } 0 < a \leq 1, & \sup(\angle BAC + \angle CAD) = 180^\circ + \cos^{-1}\left(\dfrac{a+3}{4}\right) \quad (\text{when } BC \to 3 + a) \\ (\text{If, } 0 < a < 1, \text{ The quadrilateral } ABCD \text{ is concave.}) \end{cases}$$

That is, if and only if "$1 < a < 7$,$\angle BAD$ has absolute maximum $\Longleftrightarrow BC = 4, CD = 3$" is true.

Graphs of the functions of length of BC and $\angle BAD / (\angle BAC + \angle CAD)$