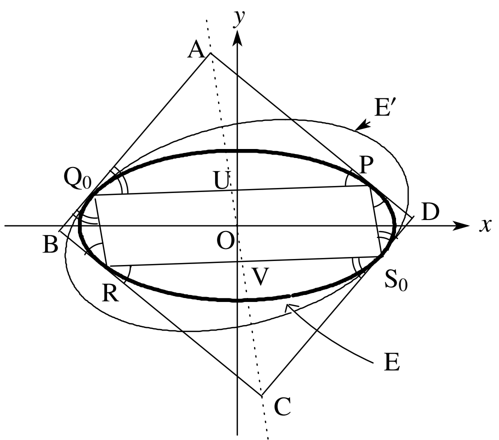
$\boxed{1}$ (1)

$$x^2 + y^2 = 5 \qquad \cdots ans.$$

(2) Fix the points P and R. Because the ellipse is symmetric with respect to O, it's enough to find the absolute maximun of PQ+QR. We can assume the existanse of an ellipse E' with focuses of P and R which ciecumsribes E. Let Q_0 and S_0 be points of contact of E and E'. If $Q \neq Q_0$, let ray PQ intersects E' at point T,then

$$PQ_0 + Q_0R = PT + TR = PQ + QT + TR > PQ + QR$$

Therefore,when point$Q$ moves in C,$PQ + QR$ has an absolute muximum when $Q = Q_0$.
Next,let A, B, C, D be crossing points of the tangents at P, Q_0, R, S_0. Line AB is a tangent to E',considering the symmetry



$$\begin{cases} \angle AQ_0P = \angle BQ_0R = \angle RS_0C = \angle PS_0D \\ \angle APQ_0 = \angle S_0PD = \angle CRS_0 = \angle BRQ_0 \end{cases} \qquad \cdots ①$$

The interior angles of $ABCD$ is 360° ,so because of ①

The quadrilateral$ABCD$is the rectangle. $\cdots$②

Again,because of ①

$$\begin{cases} \triangle APQ_0 \equiv \triangle CRS_0 \\ \triangle DPS_0 \equiv \triangle BRQ_0 \\ \triangle APQ_0 \sim \triangle DPS_0 \end{cases}$$

Then let the scalefactor of $\triangle APQ_0$and $\triangle DPS_0$ be $k : 1$.

$$\mathrm{BQ_0 : BA} = 1 : (k + 1) = \mathrm{BR : BC},\ \mathrm{DP : DA} = 1 : (k + 1) = \mathrm{DS_0 : DC}$$

$$\therefore\ Q_0R // AC // PS_0 \qquad \cdots ③$$

$$\therefore \begin{cases} \angle DAC = \angle DPS_0 = \angle APQ_0 \\ \angle ACB = \angle Q_0RB = \angle CRS_0 \end{cases} \qquad \cdots ④$$

The line AC intersects the segments PQ_0, RS_0 at U and$V$ respectively. Then because of ③,④

$$PU = AU,\ UQ_0 = VR = VC \text{ and} Q_0R = UV$$
$$\therefore\ PQ_0 + Q_0R = PU + UQ_0 + Q_0R = AU + UV + VC = AC$$

However,because of (1) and ②, A, B, C, D are in $x^2 + y^2 = 5$, so

$$\mathrm{AC} = 2\sqrt{5}$$

Therefore,the maximun distance of round a parallelogram $PQRS$ is

$$4\sqrt{5} \qquad \cdots ans.$$