Applying math software to the real world

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To download, go to
https://dl.dropboxusercontent.com/u/7079388/atcm.pdf

Thanks To

大沢文夫著「手作り統計力学」

(“Statistical mechanics- hand made ” by Fumio Oosawa)

http://bpwakate.net/Oosawa/index.html

§ 1. Life game

Suppose 6 people are sitting around the table. Each of them is assigned a number from 1 to 6, and $6m$ gold coins ($m \geqq 5$) are distributed to them randomly at the start. Then the game starts.

They throw 2 dice -green and red- in turn , and the one who has the same number as the red dice gives a coin to the one who has the same number as the green dice. If two dice have the same number, there will be no exchange of the coins but the game will be counted. If somebody has no coins left to give, he won't go bankrupt but the game will be counted (and he can still get a coin if he wins).

What will happen to their coin distribution if they keep playing this game for a long time (say 100 times)? Please choose the answer.

(1) The discrepancy among them will be bigger.
(2) The discrepancy among them will be smaller.
(3) The discrepancy among them will be the same overall

lifegame.ggb (lifegame768-1024.ggb) Lifegame.nb

§ 2. Simulation Results

1. The discrepancy at the start is relatively small.

2. As the game progresses(approximately more than m^2 times), the discrepancy gets wider and some will experience "0 coin".

3. Further the game progresses , everybody will experience “0 coin” and the distribution of the coins with respect to all the members is ‘*Geometric distribution*’.
   ($n(E) \propto e^{-\beta E}$, *n(E)*=number of people who have *E* coins)

4. Further the game progresses, everybody will experience "being rich" and the distribution of the coins of each person with respect to time is also ‘*Geometric distribution*’.
   ($t(E) \propto e^{-\gamma E}$, *t(E)*=length of time when he or she has *E* coins)

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§ 3. Lessons learned?

When the coins were distributed by the god of randomness, discrepancy was relatively small.

But after many exchanges between players, discrepancy got bigger, instead of getting smaller.

Even if the rule is completely fair, some people will go bankrupt and some people will become rich.

But in the long run, everybody will experience being rich and poor equally. Unfortunately, however, for each person, the period of being rich will be very short, even the period of being more than average will be less than half of their life time (about $1/e=0.367\cdots$).

§ 4. Maxwell-Boltzman Distribution (Theoretical Analysis)

When many classic particles such as molecules collide and exchange energy, the energy distribution is Maxwell-Boltzmann distribution.

$$\text{Number of particles of enegy } E \quad \text{is proportional to } e^{-\frac{E}{kT}}$$

$$(T \text{ is temparature, } k \text{ is Boltzman constant})$$

The distribution of coins in the Life Game converges to Maxwell-Boltzman distribution after a very long period of time.

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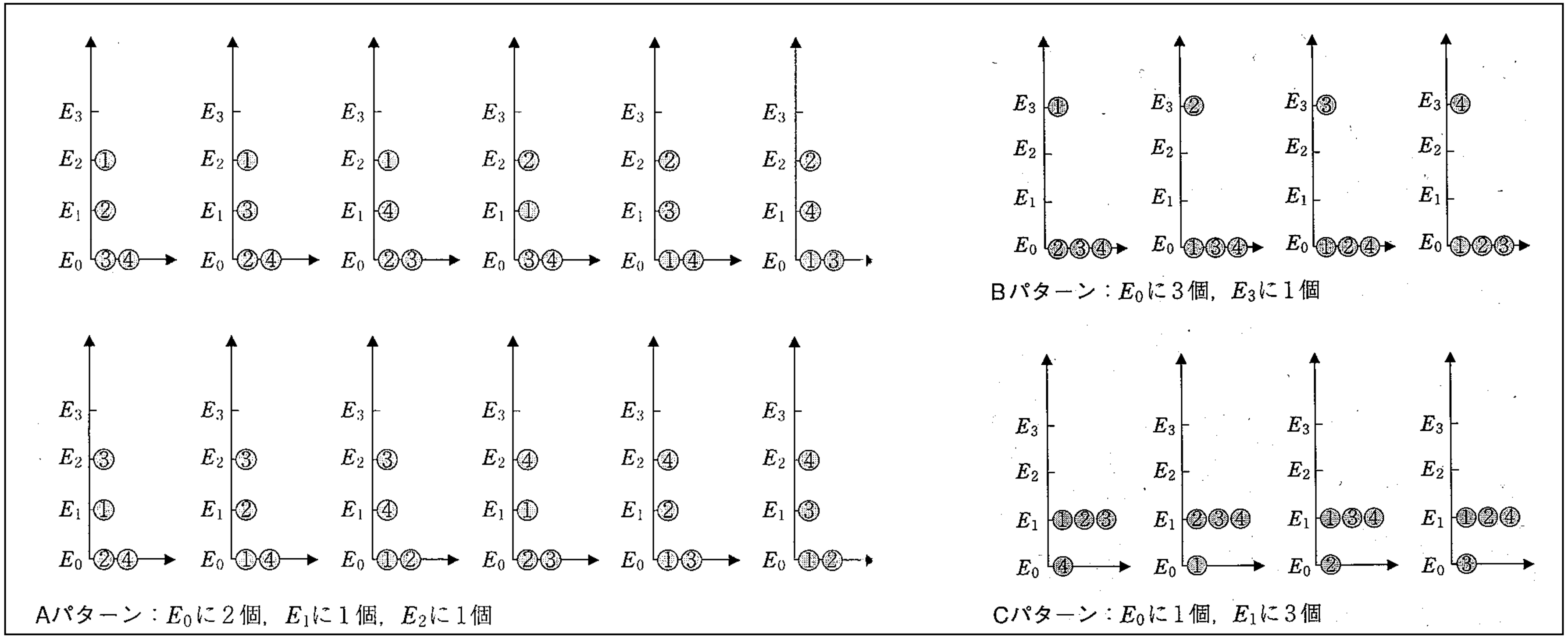
Rules of Maxwell Boltzman distribution

An exchange of 2 particles of different energy levels →different microscopic state
An exchange of 2 particles of same energy level → same microscopic state

And each and every microscopic states occurs at equal probability.
(It's called '*principle of equal a priori probabilities*')

For example, when total energy $E = 3$, number of particles $N = 4$, number of microscopic states is

$$\frac{4!}{2!1!1!} + \frac{4!}{3!1!} + \frac{4!}{3!1!} = 12 + 4 + 4 = 20 \ \left(= {}_4H_3\right)$$



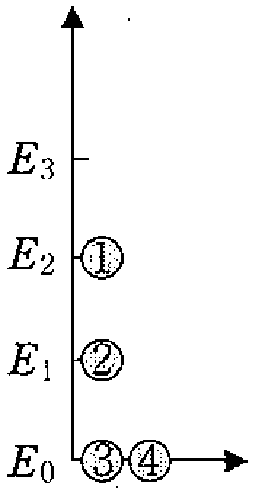
Aパターン：E_0に２個，E_1に１個，E_2に１個

Bパターン：E_0に３個，E_3に１個

Cパターン：E_0に１個，E_1に３個

Probability calculation when N=4, E=3

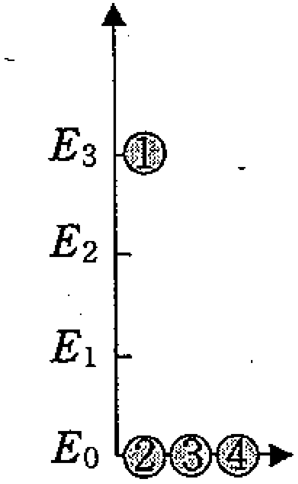
A pattern example



giver

taker	1	2	3	4
1	A	B	A	A
2	A	A	A	A
3	C	A	A	A
4	C	A	A	A

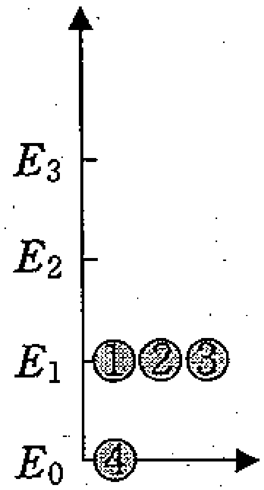
B pattern example



giver

taker	1	2	3	4
1	B	B	B	B
2	A	B	B	B
3	A	B	B	B
4	A	B	B	B

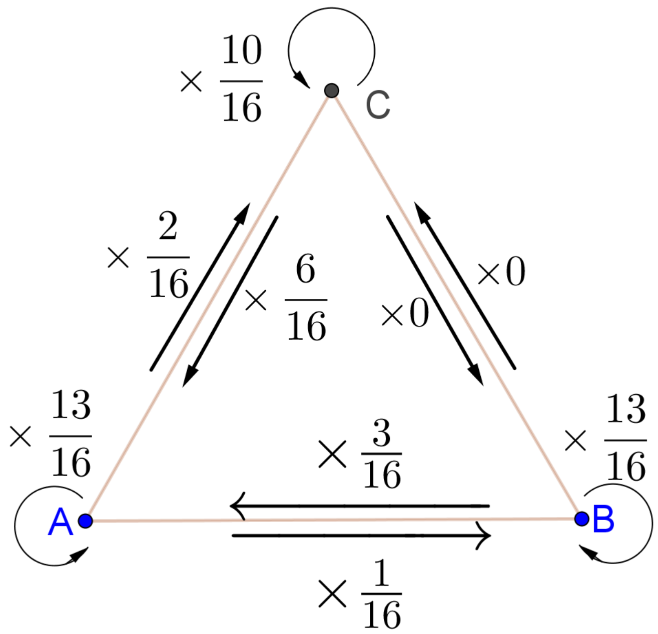
C pattern example



giver

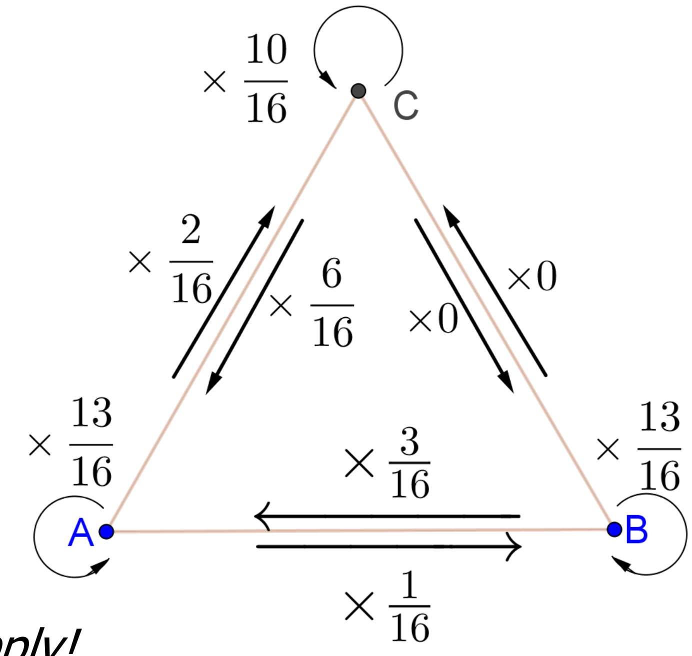
taker	1	2	3	4
1	C	A	A	C
2	A	C	A	C
3	A	A	C	C
4	C	C	C	C

Transition probability



Assume that number of states of each pattern are proportional to the numbers in the box ,for example,

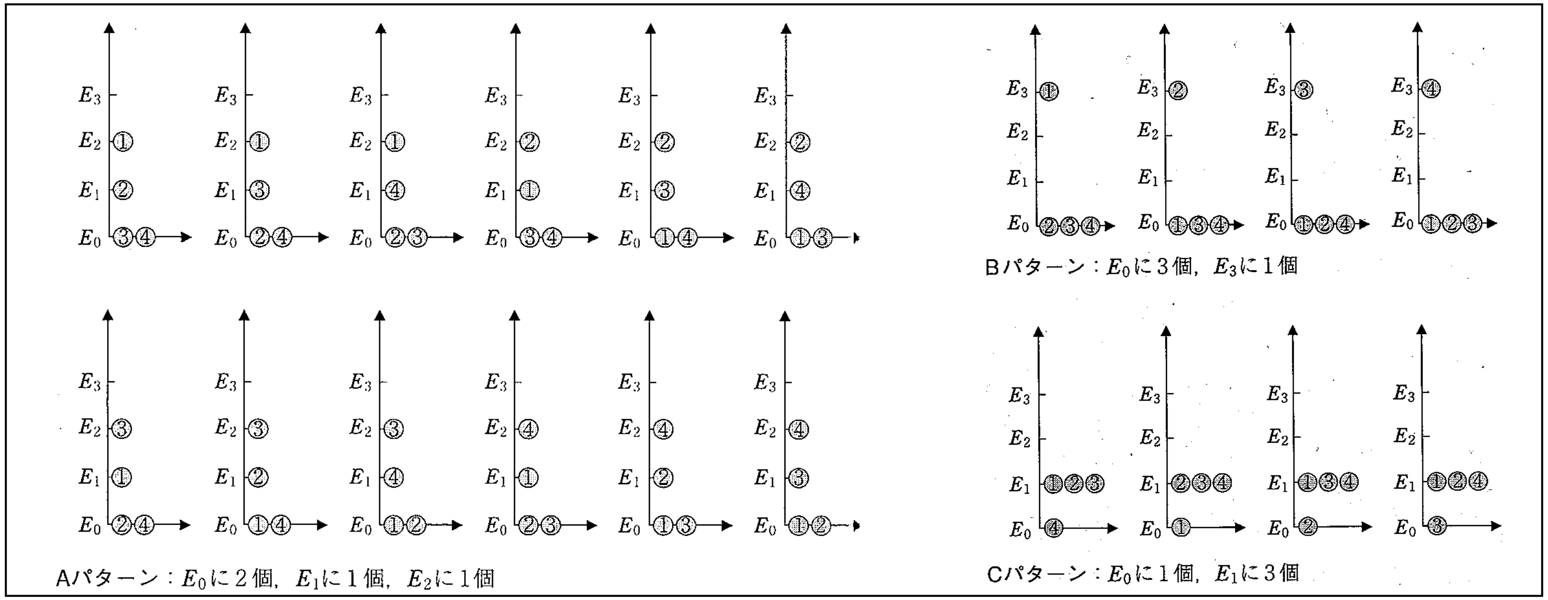
$$\begin{cases} 4\times12=48 \text{ states of pattern A,} \\ 4\times4=16 \text{ states of pattern B,} \\ 4\times4=16 \text{ states of pattern C} \end{cases}$$



Then in the next step (say, after 10000 exchanges)

$$\begin{cases} \text{number of A} \rightarrow 48\times\dfrac{13}{16}+16\times\dfrac{3}{16}+16\times\dfrac{6}{16}=39+3+6=48 \\ \text{number of B} \rightarrow 48\times\dfrac{1}{16}+16\times\dfrac{13}{16}+16\times 0=3+13=16 \\ \text{number of C} \rightarrow 48\times\dfrac{2}{16}+16\times 0+16\times\dfrac{10}{16}=6+10=16 \end{cases}$$

→ ‘*principle of equal a priori probabilities*’ *seems apply!*



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§ 5.miscellaneous

- Random Walk (2D, 3D, jungle gym)
- Monty Hall’s problem
- Central limit theorem