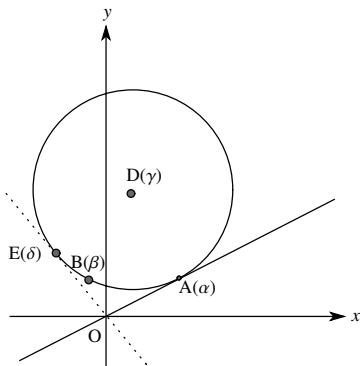


20 【解答】



$$\begin{aligned}
 (1) \quad \frac{\beta - \alpha}{0 - \alpha} &= \frac{(-1 - \sqrt{6}) + i(\sqrt{2} - \sqrt{3})}{-\sqrt{3}(\sqrt{2} + i)} \\
 &= \frac{(1 + \sqrt{6}) + i(\sqrt{3} - \sqrt{2})}{\sqrt{3}(\sqrt{2} + i)} \cdot \frac{\sqrt{2} - i}{\sqrt{2} - i} \\
 &= \frac{2}{\sqrt{3}} \left( \frac{\sqrt{3}}{2} - \frac{1}{2}i \right) \\
 &= \frac{2}{\sqrt{3}} \{ \cos(-30^\circ) + i \sin(-30^\circ) \}
 \end{aligned}$$

すなわち

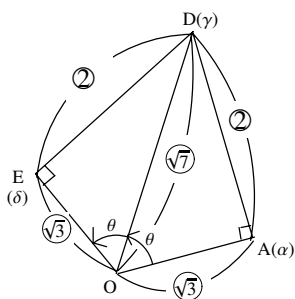
$$\begin{aligned}
 \vec{AO} &\xrightarrow[(-30^\circ) \text{ 回転}]{\text{長さを } \frac{2}{\sqrt{3}} \text{ 倍して}} \vec{AB} \text{ となるので} \\
 \angle OAB &= 30^\circ \quad \dots(\text{答})
 \end{aligned}$$

(2)  $\vec{AD} \perp \vec{AO}$  であるから, (1) より  $\angle BAD = 60^\circ$ 。ゆえに  $\triangle ABD$  は正三角形となるので、

$$\vec{AB} \xrightarrow[(-60^\circ) \text{ 回転}]{} \vec{AD}$$

よって、

$$\begin{aligned}
 \gamma - \alpha &= (\beta - \alpha) \{ \cos(-60^\circ) + i \sin(-60^\circ) \} \\
 &= \{ (-1 - \sqrt{6}) + i(\sqrt{2} - \sqrt{3}) \} \left( \frac{1 - \sqrt{3}i}{2} \right) \\
 &= -2 + 2\sqrt{2}i \\
 \gamma &= (-2 + 2\sqrt{2}i) + \alpha = (\sqrt{6} - 2) + i(2\sqrt{2} + \sqrt{3}) \quad \dots(\text{答})
 \end{aligned}$$



(3)  $\triangle OAD$  において、三平方の定理より

$$\begin{aligned}
 OA : AD : DO &= \sqrt{3} : 2 : \sqrt{7} \\
 \text{よって } \angle AOD = \theta \text{ とおくと } &\begin{cases} \sin \theta = \frac{2}{\sqrt{7}} \\ \cos \theta = \frac{\sqrt{3}}{\sqrt{7}} \end{cases}
 \end{aligned}$$

$\triangle OAD \equiv \triangle OED$  より

$$\vec{OD} \xrightarrow[\theta \text{ 回転}]{\text{長さを } \frac{\sqrt{3}}{\sqrt{7}} \text{ 倍して}} \vec{OE} \text{ となるから}$$

$$\begin{aligned}
 \delta &= \gamma \times \frac{\sqrt{3}}{\sqrt{7}} (\cos \theta + i \sin \theta) \\
 &= \{ (\sqrt{6} - 2) + i(2\sqrt{2} + \sqrt{3}) \} \times \frac{\sqrt{3}}{\sqrt{7}} \left( \frac{\sqrt{3}}{\sqrt{7}} + i \frac{2}{\sqrt{7}} \right) \\
 &= \frac{(-\sqrt{6} - 12) + i(12\sqrt{2} - \sqrt{3})}{7} \quad \dots(\text{答})
 \end{aligned}$$