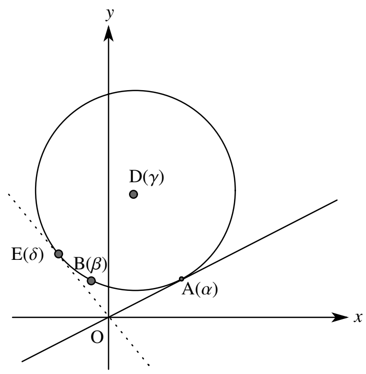
20 **【解答】**



(1)
$$\frac{\beta-\alpha}{0-\alpha}=\frac{(-1-\sqrt{6})+i(\sqrt{2}-\sqrt{3})}{-\sqrt{3}(\sqrt{2}+i)}$$
$$=\frac{(1+\sqrt{6})+i(\sqrt{3}-\sqrt{2})}{\sqrt{3}(\sqrt{2}+i)}\cdot\frac{\sqrt{2}-i}{\sqrt{2}-i}$$
$$=\frac{2}{\sqrt{3}}\left(\frac{\sqrt{3}}{2}-\frac{1}{2}i\right)$$
$$=\frac{2}{\sqrt{3}}\{\cos(-30^\circ)+i\sin(-30^\circ)\}$$

すなわち

$$\overrightarrow{\mathrm{AO}}\xrightarrow[(-30^\circ)\text{ 回転}]{\text{長さを}\frac{2}{\sqrt{3}}\text{倍して}}\overrightarrow{\mathrm{AB}}$$ となるので

$\angle\mathrm{OAB}=30^\circ$ …(答)

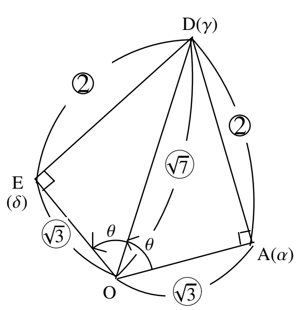
(2) $\overrightarrow{\mathrm{AD}}\perp\overrightarrow{\mathrm{AO}}$ であるから, (1) より $\angle\mathrm{BAD}=60^\circ$。ゆえに $\triangle\mathrm{ABD}$ は正三角形となるので、

$$\overrightarrow{\mathrm{AB}}\xrightarrow[(-60^\circ)\text{ 回転}]{}\overrightarrow{\mathrm{AD}}$$

よって、

$$\gamma-\alpha=(\beta-\alpha)\{\cos(-60^\circ)+i\sin(-60^\circ)\}$$
$$=\left\{(-1-\sqrt{6})+i(\sqrt{2}-\sqrt{3})\right\}\left(\frac{1-\sqrt{3}i}{2}\right)$$
$$=-2+2\sqrt{2}i$$

$\therefore\quad \gamma=(-2+2\sqrt{2}i)+\alpha=(\sqrt{6}-2)+i(2\sqrt{2}+\sqrt{3})$ …(答)



(3) $\triangle\mathrm{OAD}$ において, 三平方の定理より

$$\mathrm{OA}:\mathrm{AD}:\mathrm{DO}=\sqrt{3}:2:\sqrt{7}$$

よって $\angle\mathrm{AOD}=\theta$ とおくと
$$\begin{cases}\sin\theta=\dfrac{2}{\sqrt{7}}\\[2mm]\cos\theta=\dfrac{\sqrt{3}}{\sqrt{7}}\end{cases}$$

$\triangle\mathrm{OAD}\equiv\triangle\mathrm{OED}$ より

$$\overrightarrow{\mathrm{OD}}\xrightarrow[\theta\text{回転}]{\text{長さを}\frac{\sqrt{3}}{\sqrt{7}}\text{倍して}}\overrightarrow{\mathrm{OE}}$$ となるから

$$\delta=\gamma\times\frac{\sqrt{3}}{\sqrt{7}}(\cos\theta+i\sin\theta)$$
$$=\left\{(\sqrt{6}-2)+i(2\sqrt{2}+\sqrt{3})\right\}\times\frac{\sqrt{3}}{\sqrt{7}}\left(\frac{\sqrt{3}}{\sqrt{7}}+i\frac{2}{\sqrt{7}}\right)$$
$$=\frac{(-\sqrt{6}-12)+i(12\sqrt{2}-\sqrt{3})}{7}$$ …(答)