14 **【解答】**(1) 対称性より、4 次方程式の解は $\alpha, \beta, \gamma = -\alpha, \delta = -\beta$ であるから

$$
\begin{aligned}
x^4 + ax^3 + bx^2 + cx + d &= (x-\alpha)(x-\beta)(x+\alpha)(x+\beta) \\
&= (x^2-\alpha^2)(x^2-\beta^2) \\
&= x^4 - (\alpha^2+\beta^2)x^2 + \alpha^2\beta^2 \qquad \cdots①
\end{aligned}
$$

係数を比べて

$$a = c = 0 \qquad \cdots(答)$$

(2) ① より $b = -(\alpha^2+\beta^2),\ d = \alpha^2\beta^2$ であるから

$$
\begin{aligned}
b^2 = 3d &\iff (\alpha^2+\beta^2)^2 = 3\alpha^2\beta^2 \\
&\iff \alpha^4 - \alpha^2\beta^2 + \beta^4 = 0 \\
&\iff \left(\frac{\beta}{\alpha}\right)^4 - \left(\frac{\beta}{\alpha}\right)^2 + 1 = 0 \\
&\iff \left(\frac{\beta}{\alpha}\right)^2 = \frac{1 \pm \sqrt{3}i}{2} = \cos(\pm 60^\circ) + i\sin(\pm 60^\circ)
\end{aligned}
$$

$\beta = \alpha(\cos\theta + i\sin\theta)\ (0 < \theta < 180^\circ)$ とおくと

$$
\begin{aligned}
\cos 2\theta + i\sin 2\theta &= \cos(\pm 60^\circ) + i\sin(\pm 60^\circ) \\
\therefore \quad \theta &= 30^\circ,\ 150^\circ
\end{aligned}
$$

したがって

$$
\left.
\begin{array}{ll}
\text{2 本の対角線のなす角は} & 30^\circ \\
\text{また、線分 } AB \text{ の長さは} & \dfrac{\sqrt{6} \pm \sqrt{2}}{2}
\end{array}
\right\} \qquad \cdots(答)
$$