

Linear Transformation with CG & animation

Ogose Shigeki

Kawai-Juku

Tokyo, Japan

<http://mixedmoss.com/atcm/2012/>

1. Advantages of using Computer Graphics (CG).

- Grids can be drawn easily.
- Effects of changing the 'original objects' or 'matrix' can be seen immediately.
- Exotic objects such as photos can be transformed.
- Animations can be used.

ex1. Transformation by $\begin{pmatrix} 2 & -1 \\ 1 & 1 \end{pmatrix}$ Example of linearity $2\overline{OA} + 3\overline{OB} \rightarrow 2\overline{OA'} + 3\overline{OB'}$

[muffine.jar](#)

Transformation set

Fill in numbers or functions in the boxes or use the palette.
 Trigonometry functions by RAD: Use $\sin()$, $\cos()$, $\tan()$, "pi" for pi.
 Trigonometry functions by DEGREE: Use $\sin()$, $\cos()$, $\tan()$.

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 2.0 & -1.0 \\ 1.0 & 1.0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

Transform Default Palette Open

Advanced features: No show

Choose an object from the menubar and draw. original

When you finish drawing, choose [transform] from the menubar. transformed

ex2. Transformation of a photo by $R(\theta) = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$

[muffine.jar](#)

Transformation set

Fill in numbers or functions in the boxes or use the palette.
 Trigonometry functions by RAD: Use $\sin()$, $\cos()$, $\tan()$, "pi" for pi.
 Trigonometry functions by DEGREE: Use $\sin()$, $\cos()$, $\tan()$.

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos(30) & -\sin(30) \\ \sin(30) & \cos(30) \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

Transform Default Palette Open

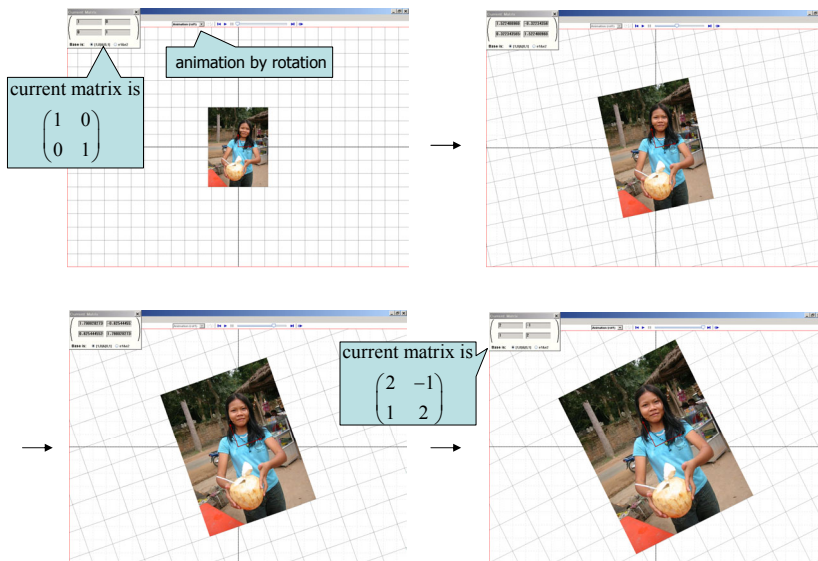
Advanced features: No show

Use a palette or type it in the field.

Choose an object from the menubar and draw. original

When you finish drawing, choose [transform] from the menubar. transformed

ex3. Animation of Rotation&Enlargement by $F = \begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix}$



Current matrix is right-at-the-moment matrix. Which starts from the unit matrix and finishes as the target matrix.

2. EigenVectors & Animation

Animation is useful for rotation - which has no real eigenvectors- ,
but it works even better for transformations which have real eigenvectors.

Next example has 2 eigenvectors.

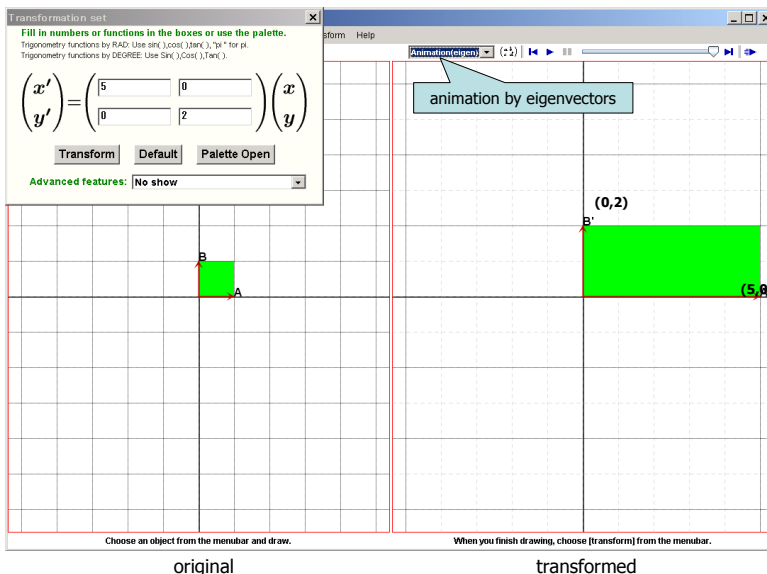
ex4. Comparison of 2 transformations which have same eigenvalues.

For $A = \begin{pmatrix} 5 & 0 \\ 0 & 2 \end{pmatrix}$, $A \begin{pmatrix} 1 \\ 0 \end{pmatrix} = 5 \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $A \begin{pmatrix} 0 \\ 1 \end{pmatrix} = 2 \begin{pmatrix} 0 \\ 1 \end{pmatrix}$
 eigenvectors : $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ & $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$, eigenvalues : 5 & 2, respectively.

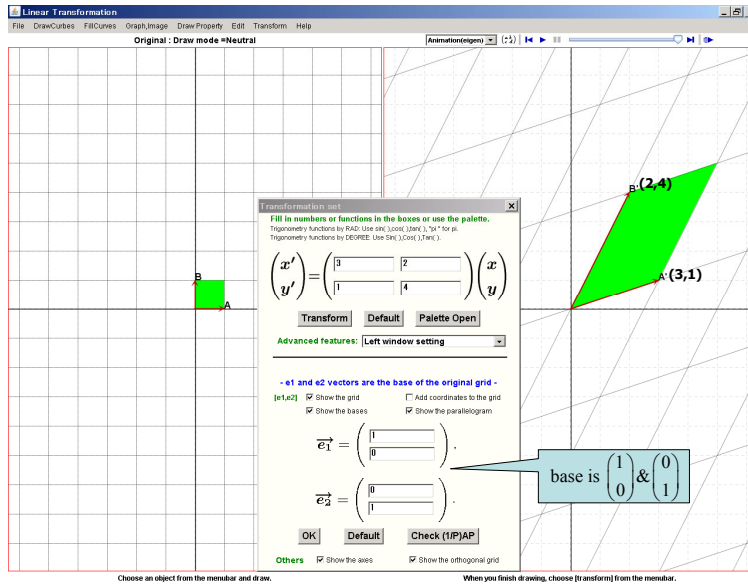
For $B = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$, $B \begin{pmatrix} 1 \\ 1 \end{pmatrix} = 5 \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $B \begin{pmatrix} -2 \\ 1 \end{pmatrix} = 2 \begin{pmatrix} -2 \\ 1 \end{pmatrix}$
 eigenvectors : $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ & $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$, eigenvalues : 5 & 2, respectively.

Transformation by $A = \begin{pmatrix} 5 & 0 \\ 0 & 2 \end{pmatrix}$. When the base is $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

muffine.jar

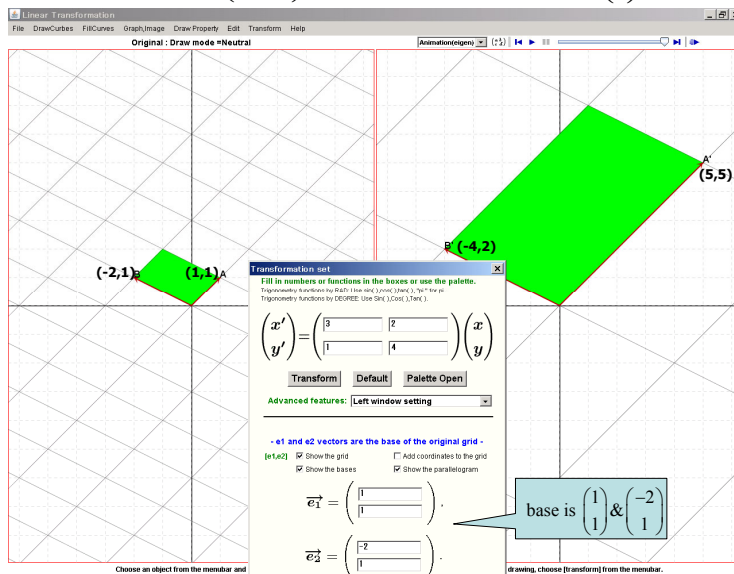


Transformation by $B = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$. When the base is $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ muffine.jar



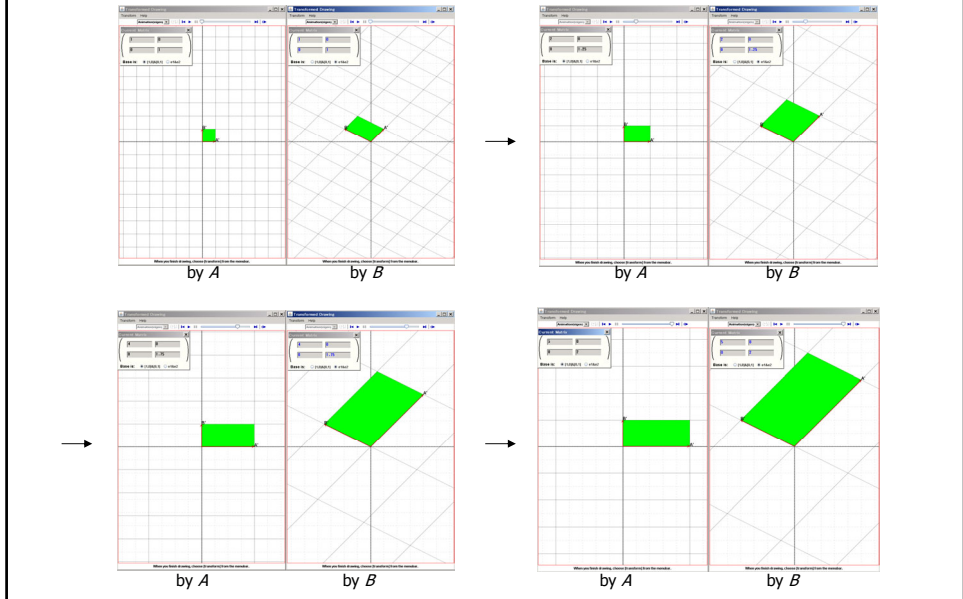
When the base is $(1,0)$ and $(0,1)$, transformation looks one of many.

Transformation by $B = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$. When the base is $\vec{e}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $\vec{e}_2 = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$



When the base is eigenvectors, transformation is easier to understand.

A and B have the same eigenvalues, thus they work similarly. Eigenvectors & values decide how linear transformations work.



$P^{-1}BP$ shows the similarity between A & B . $P^{-1}BP$ represents the same transformation as B , but it's the expression by \vec{e}_1 and \vec{e}_2 .

Transformation set

Fill in numbers or functions in the boxes or use the palette.
Trigonometry functions by RAD: Use sin(), cos(), tan(), "pi" for pi.
 Trigonometry functions by DEGREE: Use Sin(), Cos(), Tan().

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 3.0 & 2.0 \\ 1.0 & 4.0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

Transform Default Palette Open

Advanced features: (1/P)AP

Choose (1/P)AP - calculation of P⁻¹AP -

Suppose a matrix P by the base [e₁, e₂].

$$P = \begin{pmatrix} \vec{e}_1 & \vec{e}_2 \end{pmatrix} = \begin{pmatrix} 1.0 & -2.0 \\ 1.0 & 1.0 \end{pmatrix}$$

Let A be the matrix of the given transformation. Then ,

$$P^{-1}AP = \begin{pmatrix} 5.0 & 0.0 \\ 0.0 & 2.0 \end{pmatrix}$$

Change the base

Animation(eigen)

Current Matrix

$$\begin{pmatrix} 5 & 0 \\ 0 & 2 \end{pmatrix}$$

Base is: [1,0]&[0,1] e1&e2

Choose the base here. e1&e2 are the base set by you. (left)

When you finish drawing, choose [transform] from the menubar.