Linear Transformation with CG & animation

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1. Advantages of using Computer Graphics (CG).

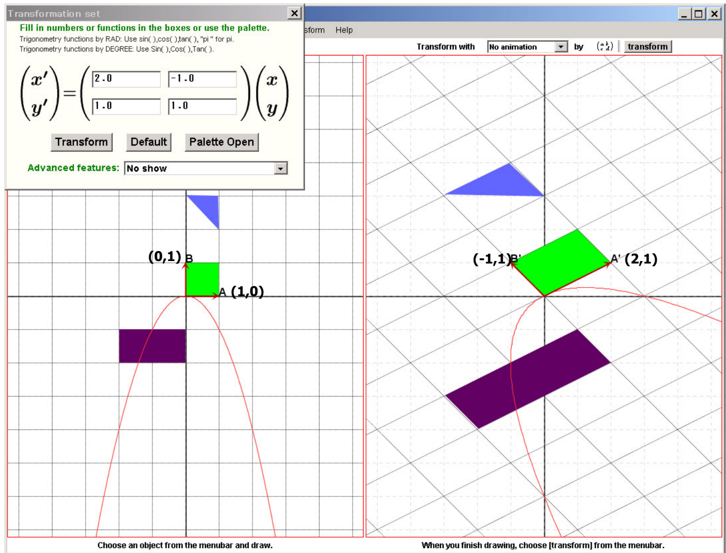
- Grids can be drawn easily.
- Effects of changing the ‘original objects’ or ‘matrix’ can be seen immediately.
- Exotic objects such as photos can be transformed.
- Animations can be used.

ex1. Transformation by $\begin{pmatrix} 2 & -1 \\ 1 & 1 \end{pmatrix}$

Example of linearity

$2\overrightarrow{OA} + 3\overrightarrow{OB} \to 2\overrightarrow{OA'} + 3\overrightarrow{OB'}$

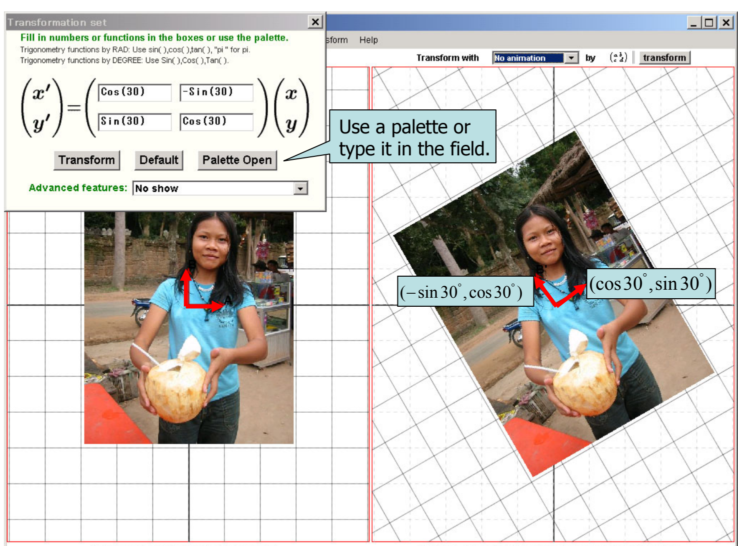
muffine.jar



original transformed

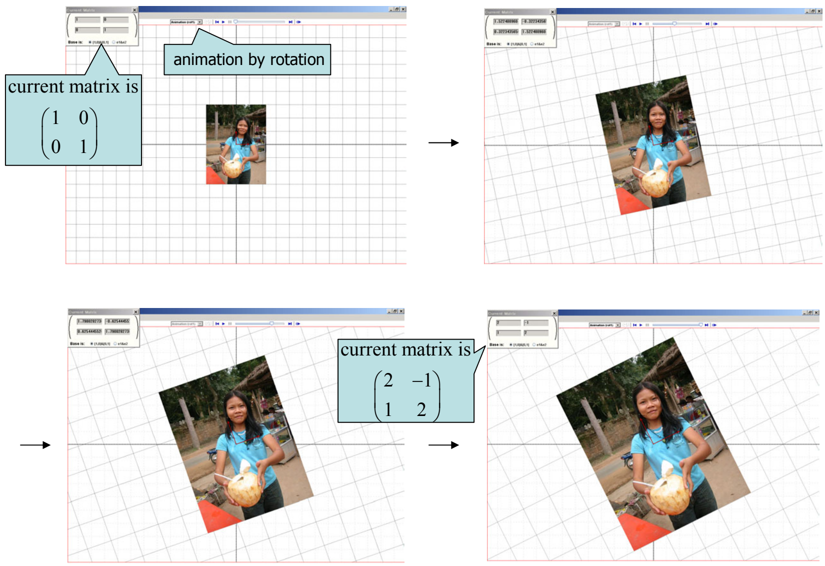
ex2. Transformation of a photo by $R(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

muffine.jar



original transformed

ex3. Animation of Rotation&Enlargement by $F = \begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix}$



Current matrix is right-at-the-moment matrix. Which starts from the unit matrix and finishes as the target matrix.

2. EigenVectors & Animation

Animation is useful for rotation - which has no real eigenvectors- ,
but it works even better for transformations which have real eigenvectors.

Next example has 2 eigenvectors.

ex4. Comparison of 2 transformations which have same eigenvalues.

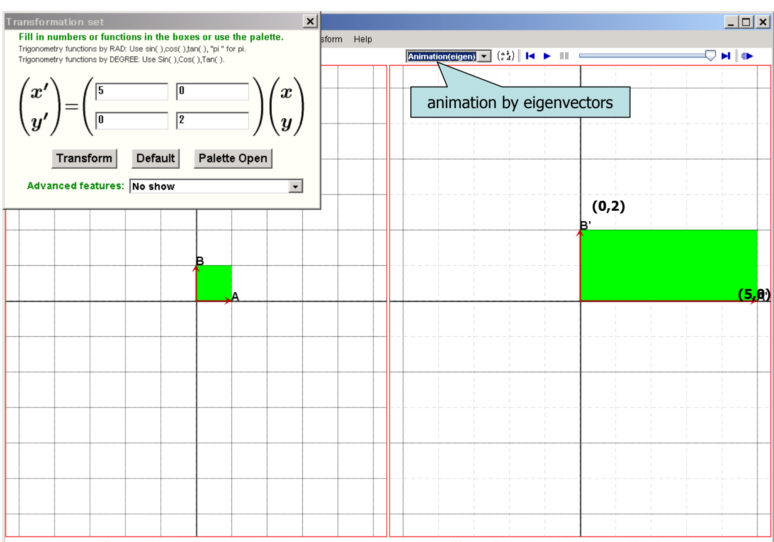
For $A=\begin{pmatrix}5 & 0\\ 0 & 2\end{pmatrix}$, $\quad A\begin{pmatrix}1\\0\end{pmatrix}=5\begin{pmatrix}1\\0\end{pmatrix}$, $A\begin{pmatrix}0\\1\end{pmatrix}=2\begin{pmatrix}0\\1\end{pmatrix}$

eigenvectors : $\begin{pmatrix}1\\0\end{pmatrix}$ & $\begin{pmatrix}0\\1\end{pmatrix}$, eigenvalues :5&2 , respectively.

For $B=\begin{pmatrix}3 & 2\\ 1 & 4\end{pmatrix}$, $\quad B\begin{pmatrix}1\\1\end{pmatrix}=5\cdot\begin{pmatrix}1\\1\end{pmatrix}$, $B\begin{pmatrix}-2\\1\end{pmatrix}=2\cdot\begin{pmatrix}-2\\1\end{pmatrix}$

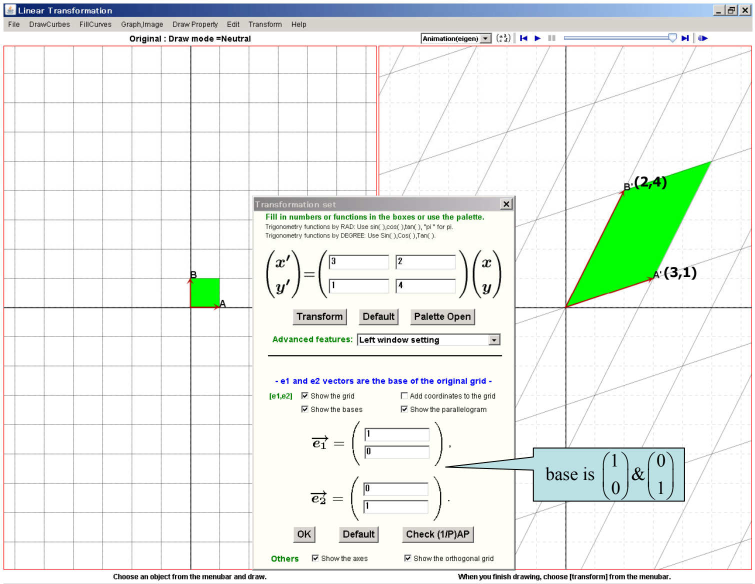
eigenvectors : $\begin{pmatrix}1\\1\end{pmatrix}$ & $\begin{pmatrix}-2\\1\end{pmatrix}$, eigenvalues :5&2 , respectively.

Transformation by $A=\begin{pmatrix}5 & 0\\ 0 & 2\end{pmatrix}$. When the base is $\begin{pmatrix}1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix}$ muffine.jar



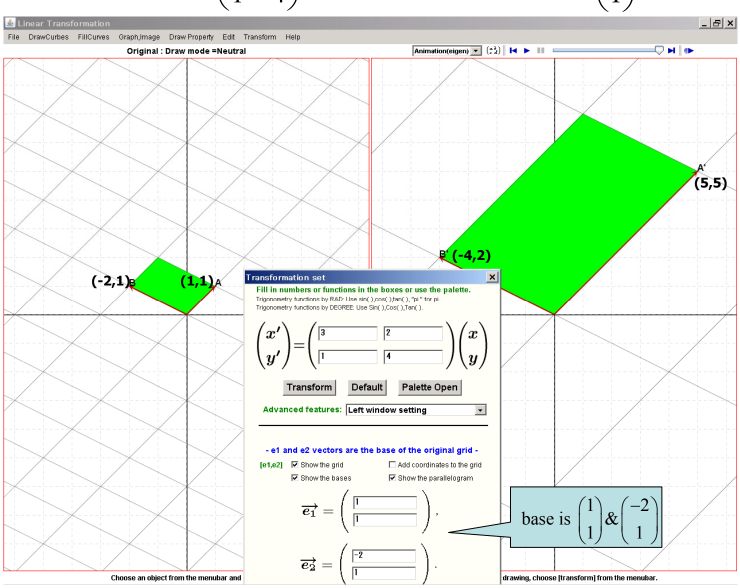
original transformed

Transformation by $B = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$. When the base is $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ muffine.jar



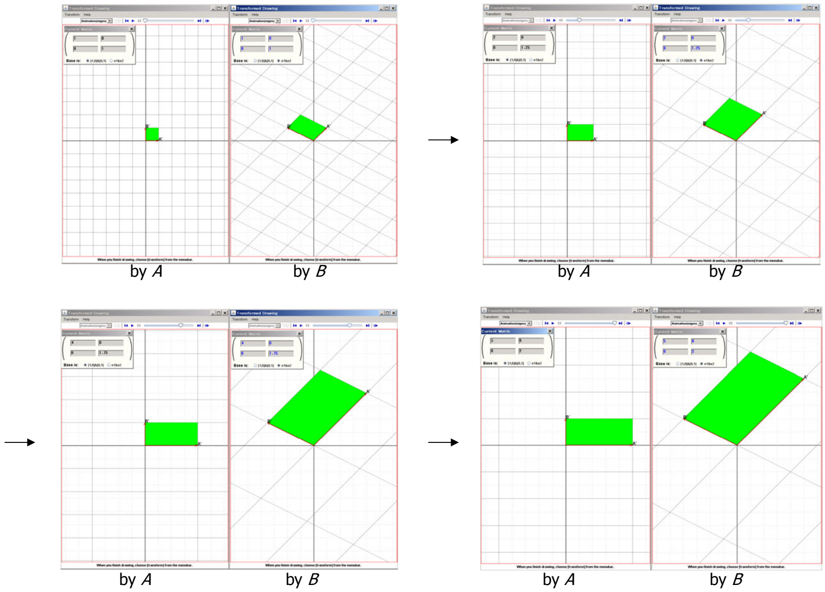
When the base is (1,0) and (0,1), transformation looks one of many.

Transformation by $B = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$. When the base is $\vec{e_1} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $\vec{e_2} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$

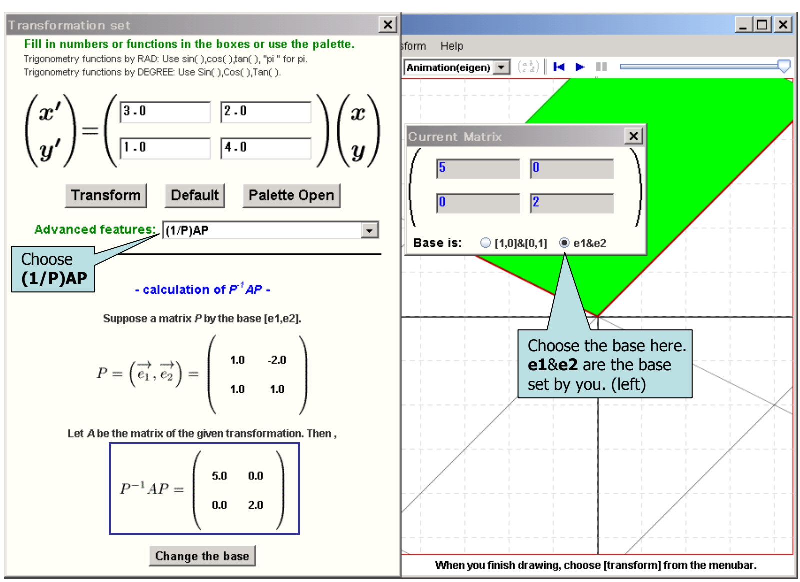


When the base is eigenvectors, transformation is easier to understand.

A and B have the same eigenvalues, thus they work similarly.
Eigenvectors&values decide how linear transformations work.



$P^{-1}BP$ shows the similarity between A&B. $P^{-1}BP$ represents the same transformation as B, but it's the expression by $\vec{e_1}$ and $\vec{e_2}$.



$$P = (\vec{e_1}, \vec{e_2}) = \begin{pmatrix} 1.0 & -2.0 \\ 1.0 & 1.0 \end{pmatrix}$$

$$P^{-1}AP = \begin{pmatrix} 5.0 & 0.0 \\ 0.0 & 2.0 \end{pmatrix}$$